The fractal dimension of small clusters

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Recent analyses of the fragment yields from the break up of excited nuclei have provided a measure of the phase diagram of nuclear matter [1]. Central to that analysis was Fisher's droplet model which describes the condensation of clusters or droplets in a low density vapor [2]. Fisher's model has also been shown to describe the cluster yields in both geometrical cluster models (percolation [4]) and thermal models (the Ising model [3]).

At liquid-vapor coexistence Fisher's model gives the yield of clusters with surface area s at a given temperature T as

$$n_s(T) \,\square\, g(s)\exp(-ws/T) \qquad (1)$$

where $g(s)$ is proportional to the degeneracy of clusters of a given surface area and w is the surface tension and the second factor is the Boltzmann factor associated with the surface energy of the cluster.

From a study of the combinatorics of lattice gas clusters in two dimensions Fisher suggested that $g(s)$ would be the number of polygons of perimeter s given by

$$g(s) \,\square\, s^{-x}\exp(\square s) \qquad (2)$$

where x is some general exponent set by the Euclidian dimension and □ is the surface entropy tension. A direct counting of such in Fig. 1 bears out Fisher's estimate.

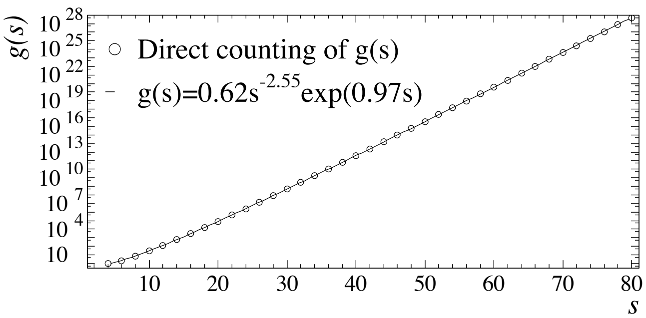

FIG. 1: Degeneracy factor for polygons on the square lattice.

Clusters can also be counted by their number of constituents A to give $g(s,A)$. The relationship between A and s can be studied from the most probable s at for a given value of A at a given temperature from $g(s,A)\exp(-ws/T)$. For compact cluster it is expected that $s \,\square\, A^{d-1/d}$ where d is the Euclidian dimension while for more diffuse clusters $s \,\square\, A^{\square}$ where □ is some exponent relating the cluster's surface to its volume. Figure 2 shows the behavior of $s(A,T)$ using the polygon combinatoric and the Boltzmann factor.

Fitting the most probable $s(A,T)$ to $a_0(T)A^{\square(T)}$ gives the effective surface to volume ration for the clusters as a function of temperature. Figure 3 shows $\square(T)$ and the fractal dimension of the cluster $D_F(T) = 1/\square(T)$ [4].

At low temperatures the clusters are most compact and $\square \sim 1/2$ while the fractal dimension is approximately equal

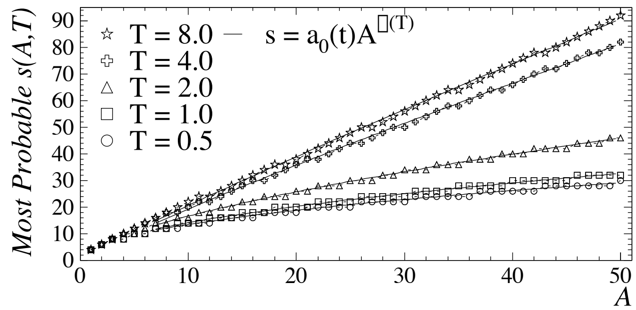

FIG. 2: The most probable surface as a function of cluster number and temperature.

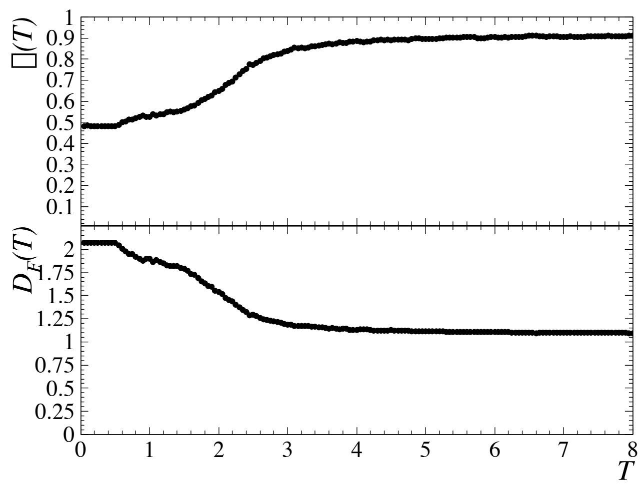

FIG. 3: The effective surface to volume exponent ($\square(T)$, top) and fractal dimension ($D_F(T)$, bottom) of small clusters as a function of temperature.

to the Euclidian dimension. As the temperature increases the value of □ increases and the fractal dimension decreases. This signals an increasing contribution to the"surface" of the cluster from its volume. At the highest temperatures the value of □ and the fractal dimension are near unity indicating that the most probable configuration for a cluster is that of a "string" of A constituents.

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